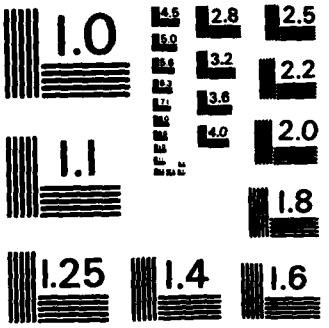
AD-A160 294 HERMITIAN AND NONNEGATIVITY PRESERVING SUBSPACES(U) 1/1
PITTSBURGH UNIV PA CENTER FOR MULTIVARIATE ANALYSIS
T MATHEW JUL 85 TR-85-25 AFOSR-TR-85-0751
UNCLASSIFIED F49620-85-C-0008 F/G 12/1 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS - 1963 - A

REPRODUCED AT GOVERNMENT EXPENSE

AFOSR-TR- 85-0751

AD-A160 294

HERMITIAN AND NONNEGATIVITY PRESERVING SUBSPACES

by

Thomas Mathew

Center for Multivariate Analysis
University of Pittsburgh
Pittsburgh, PA 15260

Center for Multivariate Analysis

University of Pittsburgh

DTIC FILE COPY

DTIC
ELECTE
OCT 15 1985
S D

Approved for public release;
distribution unlimited.

Approved for public release;
dist...

85 10 11 145

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF
This t
Dist
MATTHEW J.
Chief, Technical Information Division

HERMITIAN AND NONNEGATIVITY PRESERVING SUBSPACES*

by

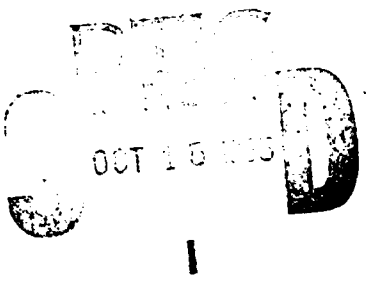
Thomas Mathew

Center for Multivariate Analysis
University of Pittsburgh
Pittsburgh, PA 15260

July 1985

Technical Report No. 85-25

Center for Multivariate Analysis
Fifth Floor, Thackeray Hall
University of Pittsburgh
Pittsburgh, PA 15260



*Research sponsored by the Air Force Office of Scientific Research (AFSC) under Contract F49620-85-C-0008. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

This document has been approved for public release and sale; its distribution is unlimited.

HERMITIAN AND NONNEGATIVITY PRESERVING SUBSPACES

by

Thomas Mathew*

Department of Mathematics and Statistics
University of Pittsburgh
Pittsburgh, PA 15260

ABSTRACT

The concepts of hermitian preserving and nonnegativity preserving subspaces of complex square matrices are introduced. Characterizations of such subspaces are obtained and applications are discussed.

Accession For
NTIS CRA&I
DTIC TAB
Unannounced
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or Special
A-1

*On leave from the Indian Statistical Instutute. Research sponsored by the Air Force Office of Scientific Research under Contract F49620-85-C-0008.

1. INTRODUCTION

We introduce the concepts of hermitian preserving and nonnegativity preserving subspaces of complex square matrices. These are special cases of the concepts of hermitian preserving and positive semidefinite preserving linear transformations given in dePillis [5] and later investigated by Hill[6], Bose and Mitra [4], Poluikis and Hill[12] and Barker, Hill and Haertel [1]. In the next section, we give several characterizations of hermitian preserving and nonnegativity preserving subspaces. In section 3, we use these results to investigate the existence and characterization of hermitian and nonnegative definite solutions to linear equations. Some applications are discussed in section 4. An important application in statistics, orginially investigated by Pukelsheim [13] and later by the author [9], is also given.

The following notations are used in the paper: $C^{n\times m}$ and $R^{n\times m}$ respectively denote the vector spaces of complex and real matrices of order $n\times m$. For a complex matrix A, A^* denotes its complex conjugate transpose and A', its transpose. The inner product of two complex matrices A and B of the same order is assumed to be the trace inner product; i.e. $\langle A,B\rangle = \operatorname{tr} B^*A$. The range of a linear transformation T is denoted as $\mathcal{R}(T)$. For a hermitian matrix A, A_+ and A_- denote the positive and negative parts of A respectively. A_+ is obtained from the spectral decomposition of A by deleting the negative eigenvalues and the corresponding projectors. A_- is then defined as $A_+ - A$. If a square matrix A is hermitian nonnegative definite, we denote $A \geq 0$.

2. HERMITIAN AND NONNEGATIVITY PRESERVING SUBSPACES

Let $\mathcal{A}$ be a subspace of $C^{n\times n}$ and let P denote the orthogonal projector onto $\mathcal{A}$ (where orthogonality is with respect to the trace inner product).

DEFINITION (1) A is said to be hermitian preserving if $P(A)$ is hermitian for every hermitian matrix A.

(2) A is said to be nonnegativity preserving if $P(A) \geq 0$ for every $A \geq 0$.

We notice that nonnegativity preserving subspaces are those which preserve the Loewner ordering, i.e. if A and B are hermitian matrices, then A is nonnegativity preserving iff $A-B \geq 0$ implies $P(A) - P(B) \geq 0$.

THEOREM 1. A is hermitian preserving iff anyone of the following equivalent conditions holds:

(i) $A^* \in A$ whenever $A \in A$

(ii) $P(A^*) = P(A)^*$ for every $A \in C^{n\times n}$.

Proof: (i) Let $A^* \in A$, for every $A \in A$. Let B be a hermitian matrix and suppose $P(B) \neq P(B)^*$. Write $P(B) = P$. Then $||B-P||^2 = ||B - \frac{P+P^*}{2}||^2 + ||\frac{P+P^*}{2} - P||^2 + \text{tr}\,(\frac{P+P^*}{2} - P)^*(B - \frac{P+P^*}{2}) + \text{tr}\,(B - \frac{P+P^*}{2})^*(\frac{P+P^*}{2} - P)$. Using the fact that B is hermitian, the last two terms simplify to $\frac{1}{2}\text{tr}\,(B - \frac{P+P^*}{2})(P-P^*) + \frac{1}{2}\text{tr}\,(B - \frac{P+P^*}{2})(P^*-P)$, which is zero. Thus we get the contradiction $||B-P||^2 > ||B - \frac{P+P^*}{2}||^2$. Conversely, let A be hermitian preserving and let $A \in A$. Suppose $A^* \notin A$. Since $A+A^*$ is hermitian, $P = P(\frac{A+A^*}{2})$ is also hermitian. Then $||A - \frac{A+A^*}{2}||^2 = ||A-P||^2 + ||P - \frac{A+A^*}{2}||^2 + \text{tr}\,(A-P)^*(P - \frac{A+A^*}{2}) + \text{tr}\,(P - \frac{A+A^*}{2})^*(A-P)$. The last two terms are zeros since $A-P \in A$ and $P - \frac{A+A^*}{2} \in A^{\perp}$ (the orthogonal complement of A). Thus we get $||A - \frac{A+A^*}{2}||^2 > ||A-P||^2$, which is a contradiction since $\frac{A+A^*}{2}$ is the hermitian matrix closest to A; see Marshall and Olkin ([8], p. 264).

(ii) If $P(A^*) = P(A)^*$ for every $A \in C^{n\times n}$, it follows that $P(A)$ is hermitian whenever A is. To prove the converse, write $A = \frac{A+A^*}{2} - i\,\frac{i(A-A^*)}{2}$ ($i = \sqrt{-1}$). Since $\frac{A+A^*}{2}$ and $\frac{i(A-A^*)}{2}$ are hermitian matrices and since $A^* = \frac{A+A^*}{2} + i\,\frac{i(A-A^*)}{2}$, it follows that $P(A^*) = P(A)^*$ when A is hermitian preserving. □

The 'if' part in part (ii) of Theorem 1 also follows from the lemma in Hill ([6], p. 260).

COROLLARY 1. A hermitian preserving subspace has a basis consisting of hermitian matrices.

THEOREM 2. Let $A_1, A_2, \ldots, A_s$ form an orthonormal basis for $\mathcal{A}$. Let a_{jk}^h denote the $(jk)^{th}$ element of A_h $(h = 1,2,\ldots,s;\ j,k = 1,2,\ldots,n)$. Then $\mathcal{A}$ is hermitian preserving iff anyone of the following equivalent conditions holds:

(i) The matrices $\sum_{h=1}^{s} \bar{a}_{jj}^h A_h$, $\sum_{h=1}^{s} (\bar{a}_{kj}^h + \bar{a}_{jk}^h) A_h$ and $\sum_{h=1}^{s} (i\,\bar{a}_{kj}^h - i\,\bar{a}_{jk}^h) A_h$ are hermitian for $j,k = 1,2,\ldots,n$.

(ii) $\sum_{h=1}^{s} A_h^* \otimes A_h$ is hermitian.

Proof: It is easy to see that the set of hermitian matrices of order n can be obtained by taking real linear combinations (linear combinations with real coefficients) of the following n^2 hermitian matrices: (a) the matrix E_{jj} having j^{th} diagonal element 1 and zeros elsewhere $(j = 1,2,\ldots,n)$ (b) the matrix F_{jk} having $(jk)^{th}$ and $(kj)^{th}$ elements 1 and zeros elsewhere $(j < k;\ j,k = 1,2,\ldots,n)$ and (c) the matrix G_{jk} having $(jk)^{th}$ element i, $(kj)^{th}$ element $-i$ and zeros elsewhere $(j < k;\ j,k = 1,2,\ldots,n)$. We have $P(B) = \sum_{h=1}^{s} (\operatorname{tr} A_h^* B) A_h$. In the place of B if we substitute the matrices E_{jj}, F_{jk} and G_{jk} defined above, we get part (i) of the theorem. It can be shown that part (i) of the theorem is equivalent to $\sum_{h=1}^{s} \bar{a}_{kj}^h a_{uv}^h = \sum_{h=1}^{s} a_{jk}^h \bar{a}_{vu}^h$ $(j,k,u,v = 1,2,\ldots,n)$, which is equivalent to $\sum_{h=1}^{s} A_h^* \otimes A_h$ being hermitian. □

Part (ii) of the above theorem also follows from Proposition 1.2 in dePillis [5] or Theorem 1 in Hill [6].

THEOREM 3. $\mathcal{A}$ is nonnegativity preserving iff (i) $\mathcal{A}$ is hermitian preserving and (ii) $A_+ \in \mathcal{A}$ for every hermitian matrix $A \in \mathcal{A}$.

Proof: Suppose A is nonnegativity preserving. If B is hermitian, we can write $B = B_+ - B_-$ and $P(B) = P(B_+) - P(B_-)$. Since $P(B_+) \geq 0$ and $P(B_-) \geq 0$, P(B) is hermitian, showing that A is hermitian preserving. Let $A \in \mathcal{A}$ be hermitian. Then we have $A_+ - A_- = A = P(A) = P(A_+) - P(A_-)$. Hence $\|A_+\|^2 + \|A_-\|^2 = \|P(A_+)\|^2 + \|P(A_-)\|^2 - 2\,\mathrm{tr}\, P(A_+)P(A_-) \leq \|P(A_+)\|^2 + \|P(A_-)\|^2$ (since $P(A_+) \geq 0$ and $P(A_-) \geq 0$). From this, it follows that $A_+ = P(A_+)$ or $A_+ \in \mathcal{A}$. Conversely, suppose (i) and (ii) in the theorem hold. Let $B \geq 0$. Then $C = P(B)$ is hermitian and $C_- \neq 0$ if C is not nonnegative definite. Now, $\|B-C\|^2 = \|B-C_+-C_-\|^2 = \|B-C_+\|^2 + \|C_-\|^2 = 2\,\mathrm{tr}\,(B-C_+)C_- > \|B-C_+\|^2$ (since $C_+C_- = 0$ and $\mathrm{tr}\, BC_+ \geq 0$). This is a contradiction since C_+ cannot be closer to B than the projection C. □

The proof of the "if" part is similar to the proof of Lemma 2 in Pukelsheim [13] and the proof of the "only if" part is similar to the proof of Lemma 1 in Mathew [9].

COROLLARY 2. A nonnegativity preserving subspace has a basis consisting of hermitian nonnegative definite matrices.

THEOREM 4. Let $A_1, A_2, \ldots, A_s$ be an orthonormal basis for $\mathcal{A}$. If $\sum_{h=1}^{s} A_h^* \otimes A_h \geq 0$, then $\mathcal{A}$ is nonnegativity preserving. This condition is also necessary if $s = 2$ or if there exists a nonsingular matrix Q such that Q^*AQ is diagonal for every hermitian matrix $A \in \mathcal{A}$.

Proof: Let A_j $(j = 1,2,\ldots,s)$ be as given in the theorem and let $A \geq 0$. Then $P(A) = \sum_{h=1}^{s} (\mathrm{tr}\, A_h^* A)A_h$ and $P(A) \geq 0$ for every $A \geq 0$ iff $\sum_{h=1}^{s} (a^* A_h^* a)A_h \geq 0$ for every vector a or equivalently

$$(a^* \otimes I)\left(\sum_{h=1}^{s} A_h^* \otimes A_h\right)(a \otimes I) \geq 0 \qquad \text{(I)}$$

for every vector a. This proves the sufficiency of the condition as asserted in the theorem. Conversely, suppose $\mathcal{A}$ is nonnegativity preserving. Then $\sum_{h=1}^{s} A_h^* \otimes A_h$ is hermitian, in view of Theorem 3(i) and Theorem 2(ii). Since $\mathcal{A}$ now has a basis con-

sisting of hermitian matrices (Corollary 1), if there exists a matrix Q as specified in the theorem, then Q^*AQ is diagonal for every $A \in \mathcal{A}$. It now follows that (I) holds for every a iff $\sum_{h=1}^{s} A_h^* \otimes A_h \geq 0$. When $s = 2$, in view of Corollary 2, let $A_1 \geq 0$, $A_2 \geq 0$ be a basis for $\mathcal{A}$. Then, using Theorem 6.2.3 in [15], we see that there exists a nonsingular matrix Q satisfying Q^*A_1Q and Q^*A_2Q are diagonal. Arguing as before, the proof can be completed. □

We observe that the condition $\sum_{h=1}^{s} A_h^* \otimes A_h \geq 0$ is similar to the condition given in Corollary 2.2 in dePillis [5]. This condition is not in general necessary for $\mathcal{A}$ to be nonnegativity preserving. For example, if $\mathcal{A}$ is the vector space of 2×2 complex matrices, then clearly, $\mathcal{A}$ is nonnegativity preserving. The matrices $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, $A_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $A_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ form an orthonormal basis for $\mathcal{A}$ and do not satisfy the condition $\sum_{h=1}^{s} A_h^* \otimes A_h \geq 0$.

REMARK 1. The existence of a nonsingular matrix Q satisfying the requirement in Theorem 4 could be verified as follows: When $\mathcal{A}$ is nonnegativity preserving, let $B_1, B_2, \ldots, B_s$ be a basis for $\mathcal{A}$ satisfying $B_j \geq 0$ $(j = 1,2,\ldots,s)$. From Theorem 3(ii) and the proof of Theorem 1(ii), it is easy to see that such a basis can be obtained. Then there exists a nonsingular matrix Q satisfying Q^*B_jQ is diagonal for $j = 1,2,\ldots,s$ iff $B_jB_0^-B_k = B_kB_0^-B_j$ for $j,k = 1,2,\ldots,s$. Here $B_0 = \sum_{j=1}^{s} B_j$ and B_0^- denotes a generalized inverse of B_0. For a proof of the above assertion, we refer to Bhimasankaram ([3], Corollary 2) or Rao and Mitra ([15], Theorem 6.5.2).

Suppose $\mathcal{A}$ is a subspace of real square matrices of order n, i.e. $\mathcal{A} \subset R^{n\times n}$. We shall call $\mathcal{A}$, a symmetry preserving subspace if $P(A)$ is symmetric whenever A is symmetric (the inner product under consideration is $\langle A,B \rangle = \text{tr } B'A$ for A,B real matrices). "Nonnegativity preserving" is defined similarly. We now give characterizations of a symmetry preserving or nonnegativity preserving subspace $\mathcal{A}$ of $R^{n\times n}$.

THEOREM 5. A subspace A of $R^{n\times n}$ is symmetry preserving iff anyone of the following equivalent conditions holds:

(i) $A' \in A$ whenever $A \in A$

(ii) $P(A') = P(A)'$ for every $A \in R^{n\times n}$.

Proof: The proofs of (i) and the 'if' part of (ii) are similar to the corresponding proofs of (i) and (ii) in Theorem 1. We shall now prove the 'only if' part of (ii). Suppose A is symmetry preserving and let $A_1, A_2, \ldots, A_s$ form an orthonormal basis for A. Then, for any $A \in R^{n\times n}$, $P(A) = \sum_{h=1}^{s} (\mathrm{tr} A_h' A) A_h$. In view of part (i) of the theorem, it follows that the matrices $A_1', \ldots, A_s'$ also form an orthonormal basis for A. Hence $P(A') = \sum_{h=1}^{s} (\mathrm{tr} A_h A') A'_h = P(A)'$. □

THEOREM 6. Let A be a subspace of $R^{n\times n}$ and let $A_1, A_2, \ldots, A_s$ form an orthonormal basis for A. Let a_{jk}^h denote the $(jk)^{th}$ element of A_h $(h = 1,2,\ldots,s;\ j,k = 1,2,\ldots,n)$. Then A is symmetry preserving iff anyone of the following equivalent conditions holds:

(i) the matrices $\sum_{h=1}^{s} a_{jj}^h A_h$ and $\sum_{h=1}^{s} (a_{kj}^h + a_{jk}^h) A_h$ are symmetric for $j,k = 1,2,\ldots,n$

(ii) $\sum_{h=1}^{s} [A_h \otimes A_h + A_h' \otimes A_h]$ is symmetric.

Proof: The matrices E_{jj} $j = 1,2,\ldots,n$ and F_{jk} $j < k$; $j,k = 1,2,\ldots,n$ defined in the proof of Theorem 2 form a basis for the subspace of real symmetric matrices. The proof of the theorem can now be completed along the lines of the proof of Theorem 2.

THEOREM 7. Let A be a subspace of $R^{n\times n}$. Then

(a) A is nonnegativity preserving iff (i) A is symmetry preserving and (ii) $A_+ \in A$ for every symmetric matrix $A \in A$

(b) If $A_1, A_2, \ldots, A_s$ are as in Theorem 6 and if $\sum_{h=1}^{s} [A_h \otimes A_h + A_h' \otimes A_h] \geq 0$, then A is nonnegativity preserving.

Proof: The rpoofs of (a) and (b) are similar to the proofs of Theorem 3 and the first part of Theorem 4 respectively. □

REMARK 2. When A is a subspace of $R^{n\times n}$, the conclusions in Corollary 1 and Corollary 2 are not true in general if we replace "hermitian" by "symmetric". This is easily seen by taking A to be $R^{n\times n}$ itself.

REMARK 3. When A is a subspace of real symmetric matrices, all the conclusions in Corollary 1, Corollary 2, Theorem 4 and Remark 1 remain valid if we replace "hermitian" by "symmetric" wherever applicable.

3. HERMITIAN AND NONNEGATIVE SOLVABILITY OF LINEAR EQUATIONS.

Let T be a linear transformation from $C^{n\times n}$ into $C^{p\times q}$. In this section, we are interested in the existence and characterization of hermitian and hermitian nonnegative definite (nnd) solutions to the linear equation $TX = Q$, where $Q \in R(T)$ (the range of T). T^* denotes the adjoint of T and T^+ denotes the maximal generalized inverse of T as defined in Ben Israel and Greville ([2], p. 318). T^+ is also the pseudoinverse of T as defined in Holmes ([7], pp. 216-226). Then $R(T^+) = R(T^*)$ and T^+T is the orthogonal projector onto $R(T^*)$ (see Theorem 2 on p. 320 in [2]). When T is a matrix, T^+ coincides with the Moore Penrose inverse of T. We say that $TX = Q$ is hermitian solvable (nonnegatively solvable) if there exists a hermitian (respectively hermitian nnd) matrix X satisfying $TX = Q$.

DEFINITION. We say that T^+ characterizes the hermitian solvability (nonnegative solvability) of $TX = Q$ if the existence of a hermitian (hermitian nnd) solution to $TX = Q$ implies that T^+Q is hermitian (respectively hermitian nnd).

Thus, if we know that T^+ characterizes the hermitian solvability (nonnegative solvability) of $TX = Q$, in order to examine if $TX = Q$ has a hermitian (hermitian nnd) solution, it is enough to check if T^+Q is hermitian (respectively hermitian nnd).

THEOREM 8. (i) For every $Q \in R(T)$, T^+ characterizes the hermitian solvability of $TX = Q$ iff $R(T^*)$ is hermitian preserving.

(ii) For every $Q \in R(T)$, T^+ characterizes the nonnegative solvability of $TX = Q$ iff $R(T^*)$ is nonnegativity preserving.

Proof: Suppose $R(T^*)$ is hermitian preserving. Then T^+TX is hermitian whenever X is hermitian. Thus when X is a hermitian solution to $TX = Q$, $T^+TX = T^+Q$ is also a hermitian solution to $TX = Q$. Conversely let T^+ characterize the hermitian solvability of $TX = Q$ for every $Q \in R(T)$. Let X_0 be any hermitian matrix and let $TX_0 = Q_0$. Then T^+Q_0 is hermitian. But $T^+Q_0 = T^+TX_0$ and X_0 is an arbitrary hermitian matrix. Hence $R(T^*)$ is hermitian preserving. This proves (i). (ii) is proved similarly. □

REMARK 4. If T is a linear transformation from $R^{n\times n}$ into $R^{p\times q}$, the conclusions in Theorem 8 remain valid if we replace "hermitian" by "symmetric". (Symmetric solvability of $TX = Q$ is defined analogous to hermitian solvability.)

4. APPLICATIONS.

(a) Let $A_j \in C^{p_j\times n}$ and $B_j \in C^{p_j\times p_j}$ for $j = 1,2,\ldots,s$. Let $\sum_{j=1}^{s} p_j = p$ and consider the system of equations $A_jXA_j^* = B_j$ $(j = 1,2,\ldots,s)$ (assumed to be consistent). Here TX is a vector consisting of the s matrices $(A_1XA_1^*,\ldots,A_sXA_s^*)$ and $R(T^*)$ consists of all complex matrices of the form $\sum_{j=1}^{s} A_j^*Z_jZ_j$, where $Z_j \in C^{p_j\times p_j}$. From Theorem 1(i), it follows that $R(T^*)$ is a hermitian preserving subspace. Hence the existence of a common hermitian solution to the above equations can be verified by examining if T^+B is hermitian, in which case T^+B itself gives a hermitian solution (B denotes $(B_1,B_2,\ldots,B_s)$).

(b) Let A_j and B_j be as defined in (a) and assume further that $B_j \geq 0$ and $R(B_j) = R(A_j)$. Let A be a matrix satisfying $R(A^*) = R(A_1^*: A_2^*:\ldots:A_s^*)$. Now consider the equations

$A_j A^* X A A_j^* = B_j$, $j = 1,2,\ldots,s$ and suppose we want to examine if the above equations admit a common hermitian positive definite solution X. This problem arises in connection with the identification problem for shorted matrices, see the proof of Theorem 1 in Mathew and Mitra [10] (see also Mitra [11] for the details of this problem). Let T denote the corresponding linear transformation (as in the case of (a) above). We notice that if X is a hermitian nnd solution to the above equations, then $P_A X P_A + (I-P_A)$ is a hermitian positive definite solution, where P_A denotes the projection matrix $A(A^*A)^-A^*$. Thus, it is enough to know if the above system of equations admits a common hermitian nnd solution. In case $R(T^*)$ is nonnegativity preserving, this can be verified by applying Theorem 8(ii).

(c) Nonnegative estimation of variance components.

Let Y be a random R^n-vector with mean vector of the form $A\beta$ and dispersion matrix $\sum_{j=1}^{k} \theta_j V_j$. Here A is a known matrix, β is a vector of unknown parameters, V_j $(j = 1,2,\ldots,k)$ are known real symmetric matrices and θ_j $(j = 1,2,\ldots,k)$ are unknown parameters known as variance components. $\theta = (\theta_1,\theta_2,\ldots,\theta_k)$ is such that $\sum_{j=1}^{k} \theta_j V_j \geq 0$. We are interested in estimating the linear parametric function $q'\theta = q_1\theta_1 + \ldots + q_k\theta_k$. Let $M = I-AA^+$ and let T be the linear transformation from $R_s^{n\times n}$ into R^k defined as $TX = (\mathrm{tr}XMV_1M,\ldots,\mathrm{tr}XMV_kM)'$, for $X \in R_s^{n\times n}$. Here $R_s^{n\times n}$ denotes the space of real symmetric matrices of order n. Then it is known that $q'\theta$ admits a translation invariant quadratic unbiased estimator iff $TX = q$ has a solution $X \in R_s^{n\times n}$, in which case $Y'MXMY$ is an estimator with the desired properties (see [14], pp. 302-305). When $q \in R(T)$, $q'\theta$ admits a nonnegative definite quadratic unbiased estimator iff $TX = q$ admits a solution $X \geq 0$. When $R(T^*)$ is nonnegativity preserving, this can be verified by examining if $T^+q \geq 0$. We notice that $R(T^*)$ is the vector space spanned by the matrices $MV_1M,\ldots,MV_kM$. Also, $Y'(T^+q)Y$ is the well known minimum norm quadratic unbiased estimator (MINQUE) of $q'\theta$, introduced by

C.R. Rao ([14], pp. 302-305). Thus, when $R(T^*)$ is nonnegativity preserving, the existence of an nnd quadratic unbiased estimator of $q'\theta$ is equivalent to the nonnegativity of its MINQUE. Further details regarding this observation are given in [13] and [9].

(d) Nonnegativity preserving subspaces of R^n

Let S be a subspace of R^n and let P denote the orthogonal projector onto S. (Here we assume that orthogonality is with respect to the inner product $(x,y) = y'x$. More general inner product can be considered and the modifications in such a case will be obvious). We say that S is nonnegativity preserving if Px has nonnegative components whenever the vector x has nonnegative components. Clearly, S is nonnegativity preserving iff the matrix P has nonnegative entries. Let $A \in R^{k\times n}$. Then, for every $b \in R(A)$, A^+b characterizes the nonnegative solvability of $Ax = b$ iff $R(A')$ is nonnegativity preserving. Thus, when $R(A')$ is nonnegativity preserving, in order to verify if $Ax = b$, $b \in R(A)$, has a solution x with nonnegative components, it is enough to examine if the vector A^+b has nonnegative components, in which case $x = A^+b$ is a desired solution.

REFERENCES

1. G.P. Barker, R.D. Hill and R.D. Haertel, On the completely positive and positive semidefinite preserving cones, Linear Algebra Appl. 56: 221-229 (1984).

2. A. Ben-Israel and T.N.E. Greville, Generalized Inverses: Theory and Applications, John Wiley and Sons, N.Y. (1974).

3. P. Bhimasankaram, Simultaneous reduction of several hermitian forms, Sankhyā, series A 33: 417-422 (1971).

4. N.K. Bose and S.K. Mitra, Nonnegative definite bihermitian forms and linear maps preserving positivity, preprint, University of Pittsburgh (1975).

5. J. de Pillis, Linear transformations which preserve hermitian and positive semidefinite transformations, Pac. J. Math. 23: 129-137 (1967).

6. R.D. Hill, Linear transformations which preserve hermitian matrices, Linear Algebra Appl. 6: 257-262 (1973).

7. R.B. Holmes, A Course on Optimization and Best Approximation, Springer, N.Y. (1972).

8. A.W. Marshall and I. Olkin, Inequalities: Theory of Majorization and Its Applications, Academic Press, N.Y. (1979).

9. T. Mathew, On nonnegative quadratic unbiased estimability of variance components, Ann. Statist. 12: 1566-1569 (1984).

10. T. Mathew and S.K. Mitra, Shorted operators and the identification problem-the real case, IEEE Trans. Circuits Syst. CAS-31: 299-300 (1984).

11. S.K. Mitra, Shorted operators and the identification problem, IEEE Trans. Circuits Syst. CAS-29: 581-583 (1982).

12. J.A. Poluikis and R.D. Hill, Completely positive and hermitian preserving linear transformations, Linear Algebra Appl. 35: 1-10 (1981).

13. F. Pukelsheim, On the existence of unbiased nonnegative estimates of variance covariance components, Ann. Statist. 9: 293-299.

14. C.R. Rao, Linear Statistical Inference and Its Applications, John Wiley and Sons, N.Y. (1973).

15. C.R. Rao and S.K. Mitra, Generalized Inverse of Matrices and Its Applications, John Wiley and Sons, N.Y. (1971).

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 85-25	2. GOVT ACCESSION NO: A160 294
3. RECIPIENT'S CATALOG NUMBER:	
4. TITLE (and Subtitle): Hermitian and Nonnegativity Preserving Subspaces	5. TYPE OF REPORT & PERIOD COVERED: Technical - July 1985
	6. PERFORMING ORG. REPORT NUMBER: 85-25
7. AUTHOR(s): Thomas Mathew	8. CONTRACT OR GRANT NUMBER(s): F49620-85-C-0008
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Center for Multivariate Analysis, 515 Thackeray Hall, University of Pittsburgh, Pittsburgh, PA 15260	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F, 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research, Department of the Air Force, Bolling Air Force DC 20332	12. REPORT DATE: July 1985
	13. NUMBER OF PAGES: 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19 KEY WORDS (Continue on reverse side if necessary and identify by block number)

Hermitian preserving subspace, symmetry preserving subspace, nonnegativity preserving subspace.

20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

The concepts of Hermitian preserving and nonnegativity preserving subspaces of complex square matrices are introduced. Characterizations of such subspaces are obtained and applications are discussed.

DD FORM 1 JAN 73 1473

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

FILMED

11-85

DTIC